Control of a Synchronous Satellite by Continuous Radial Thrust

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The motion of a satellite in an arbitrary circular orbit, resulting from the action of small, continuous forces acting circumferentially, radially, and normally to the orbit for short periods is discussed. A correlation is made between this motion and that of an equatorial synchronous satellite undergoing corrective orbital maneuvers by a single thruster, nominally pointing in a radial direction, but vectored to produce circumferential, normal, and incremental radial thrust components. A system is described employing such a single thruster for attitude and position control, and this system is shown to be competitive with a multiple-thruster system for long-life satellites.

1. Introduction

A SYNCHRONOUS satellite is one that is launched in an easterly direction into a circular orbit having the same period as the rotational period of the earth. If this orbit lies in the equatorial plane, the satellite will appear to remain fixed in the sky, relative to an observer on earth.

Over an extended period of time, the satellite, however, will not maintain its relative fixed position, owing to known perturbations from the moon, sun, and the earth's elliptical equatorial section (triaxiality). The conventional way of overcoming these perturbations is to have a multiple thruster system onboard the satellite which delivers a succession of impulses of appropriate magnitude and direction. Because of errors inherent in the control of thrust duration and thrust magnitude, and because of the impulsive nature of the corrective forces, the intermittent action of the thrusters will further perturb the position of the satellite. A simpler and presumably more reliable system to employ for position control consists of a *single thruster* (low thrust), operating continuously, and thrusting in a nominally radial direction. It is shown in this paper that a radial thruster of this type can maintain a satellite in a synchronous orbit at an altitude differing from that of the classical synchronous orbit. If the thruster is provided with the capability of vectoring in any direction about its nominal position, it will be able to correct the orbital errors caused by the sun, moon, and the earth's triaxiality. By mounting the thruster on an arm that is gimballed at the satellite center of mass, and again at a point close to the thruster, it will be possible to provide independent control of the position and attitude of the satellite.

Ion propulsion is particularly suited for attitude and position control due to the anticipated long lifetime of the satellite. In comparing a multiple (9) ion thruster system with an equivalent single ion thruster, it is found that the total weights of both systems are quite similar.

2. Effect of a Small Continuous Force on a Satellite

In order to determine the effects of the natural perturbations on a stationary satellite (i.e., a synchronous satellite in the equatorial plane) and the maneuvers needed to correct these effects, we first discuss the motion of an earth satellite, initially in an arbitrary circular orbit, under the separate action of three, orthogonal, continuous forces. Analyses of the motion have been reported elsewhere,[1–5] but the development given here employs a common approach for all three forces. The forces are 1) circumferential force F_c, which is in the plane of the orbit and perpendicular to the radius vector from the satellite to the center of the earth; 2) normal force F_n, which acts on the satellite in a direction perpendicular to its orbital plane; and 3) radial force F_r, which lies along the radius vector to the center of the earth (F_r is not the gravitational force). The positive directions of these forces are defined in Fig. 1. We shall first consider the circumferential force F_c.

2.1 Circumferential Force

The motion of the satellite under the action of a constant acceleration $f_c = F_c/m$ (m = mass of satellite) has been derived analytically in Ref. 1. If r denotes the distance of the satellite from the center of the earth and h the angular momentum per unit mass, then, for f_c small compared with the acceleration due to gravity, it is shown that

$$r/r_0 = 1 + \lambda_c(2\tau - 2\sin\tau) + \lambda_c^2(3\tau^2 + 3\tau^2\cos\tau - \tau\sin\tau - 2 - 4\sin^2\tau + 2\cos\tau) + \ldots \quad (1)$$

$$h/h_0 = 1 + \lambda_c\tau + \lambda_c^2(\tau^2 - 2 + 2\cos\tau) + \ldots \quad (2)$$

where r_0 and h_0 are the initial orbital radius and angular momentum, respectively, and

$$\tau = 2\pi(t/T_0) \quad (3)$$

$$\lambda_c = (1/r_0)(T_0/2\pi)^2 f_c \qquad \lambda_c \ll 1 \quad (4)$$

Here, t denotes time, and T_0 is the period of the initial orbit. Note that τ is dimensionless time, equal to the true anomaly for an orbit of zero eccentricity, and λ_c is dimensionless acceleration, expressed in terms of the gravitational acceleration at distance r_0.

All of the terms in the coefficient (not shown) of λ_c^3 in Eq. (1) are of order τ^3 or less, except the term $\frac{9}{4}\tau^4 \sin\tau$. If we make the restriction $\lambda_c\tau^2 \approx 0(1)$, the terms containing λ_c^2 *and* λ_c^3 will be of order $(\lambda_c\tau)^2$, so that Eq. (1) may be written

$$r/r_0 = 1 + \lambda_c(2\tau - 2\sin\tau) + 0(\lambda_c^2\tau^2) \quad (5)$$

The true anomaly θ of the satellite (refer to Fig. 1) is given by

$$\theta = \frac{T_0}{2\pi}\int_0^\tau \frac{h}{r^2}\,d\tau$$

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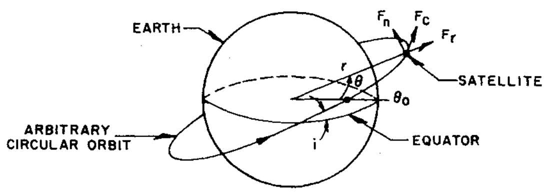

Fig. 1 Definition of F_c, F_r, F_n.

Employing the binomial expansion in Eq. (1), we have, to first order,

$$r^{-2} \approx r_0^{-2}[1 - 4\lambda_c(\tau - \sin\tau)]$$

Substituting this and Eq. (2) in the preceding integral then gives

$$\theta \approx \tau - \tfrac{3}{2}\lambda_c\tau^2 + 4\lambda_c(1 - \cos\tau) \tag{6}$$

Equations (5) and (6) then describe (to first order) the motion of the satellite under the action of the constant circumferential acceleration f_c.

Of particular interest are the changes in radial distance r, true anomaly θ, orbital period T, and orbit eccentricity e resulting from the continuous action of f_c over an interval of time, $\tau \leq 2\pi$. From Eq. (5), the change in r is

$$\Delta r = r - r_0 \approx 2\lambda_c r_0(\tau - \sin\tau) \tag{7}$$

The change in θ, due to the action of f_c, is

$$\Delta\theta = \theta - \tau \approx -\tfrac{3}{2}\lambda_c\tau^2 + 4\lambda_c(1 - \cos\tau) \tag{8}$$

After the application of f_c, the satellite orbit will be an ellipse. The period T of this ellipse is related to the period T_0 of the initial circular orbit according to

$$T/T_0 = (a/r_0)^{3/2}$$

where a is the semimajor axis of ellipse. Now,

$$a/r_0 = -(r_0/2)(f_c/\lambda_c)(1/E_T)$$

where E_T is the total energy per unit mass of satellite following the application of f_c. But, $E_T = E_0 + E$, where E_0 is the energy of the initial orbit and E is the work done on the satellite by f_c. Then, since

$$E \approx f_c r_0 \tau$$

we see that

$$a/r_0 \approx 1 + 2\lambda_c\tau$$

so that

$$\Delta T = T - T_0 \approx 3\lambda_c T_0 \tau \tag{9}$$

The eccentricity of the ellipse[1] to first order is given as

$$e \approx 4\lambda_c|\sin(\tau/2)| \tag{10}$$

2.2 Radial Force

Consider a small, constant, outwardly directed radial force F_r to be applied to a satellite that is moving initially in a circular orbit (see Fig. 2). The gravitational acceleration is

$$g = \mu/r_0^2$$

where μ is the gravitational constant of the earth. If F_r remains constant and always points along the radial direction, then we may think of a new gravitational acceleration as

$$g' = (\mu/r_0^2) - (F_r/m) = \mu'/r_0^2$$

Thus, the application of the small, constant, outward radial force F_r has the effect of reducing the earth's gravity constant (i.e., the earth's mass) to a new value given by

$$\mu' = \mu - r_0^2 f_r \tag{11}$$

where $f_r = F_r/m$. Now, the orbital velocity v_0 of the satellite in the initial, circular orbit is $v_0^2 = \mu/r_0$. In order for the satellite to move in a circular orbit during the application of F_r its orbital velocity must be $v^2 = \mu'/r_0$. However, at the instant F_r is applied, the satellite velocity is v_0, and since $v_0^2 > \mu'/r_0$, the satellite must then move along the ellipse shown in Fig. 2, where the perigee distance is equal to r_0. Note that if F_r were directed inward, then $\mu' = \mu + r_0^2 f_r$, and the ellipse would lie inside the original orbit where the apogee distance would be r_0.

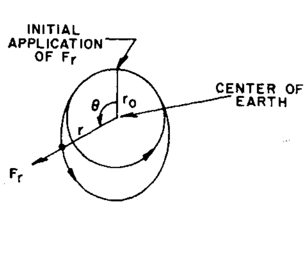

Fig. 2 Effect of small radial force on circular orbit (exaggerated).

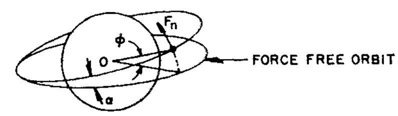

Fig. 3 Effect of small normal force on circular orbit.

We are interested in determining the changes in radial distance r, true anomaly θ, orbital period T, and orbit eccentricity e resulting from the continuous action of F_r over an interval of time $\tau \leq 2\pi$. The equation of the ellipse in Fig. 2 is

$$1/r = (\mu'/h^2)(1 + e\cos\theta)$$

where h is the angular momentum per unit mass, which is identical to that on the original force-free orbit, since the applied force is in a radial direction. If we define the dimensionless acceleration λ_r as

$$\lambda_r = \frac{1}{r_0}\left(\frac{T_0}{2\pi}\right)^2 f_r = \frac{r_0^2}{\mu} f_r \tag{12}$$

then the equation of the ellipse becomes

$$1/r = (1/r_0)(1 - \lambda_r)(1 + e\cos\theta)$$

Since $r = r_0$ when $\theta = 0$, the preceding equation gives

$$e = \lambda_r \tag{13}$$

Employing the binomial expansion, the equation of the ellipse becomes

$$\Delta r = r - r_0 \approx r_0\lambda_r(1 - \cos\theta) \approx r_0\lambda_r(1 - \cos\tau) \tag{14}$$

where τ is defined in Eq. (3)

The period of the ellipse T is related to the period of the initial orbit according to

$$T/T_0 = (\mu a^3/\mu' r_0^3)^{1/2}$$

where a is the semimajor axis of ellipse, equal to one-half the sum of the apogee ($\theta = \pi$) and perigee ($\theta = 0$) distances, and so $a = r_0(1 + \lambda_r)$. The preceding equation then yields

$$\Delta T = T - T_0 \approx 2\lambda_r T_0 \tag{15}$$

The true anomaly is given by

$$\theta = \frac{T_0}{2\pi}\int_0^\tau \frac{h}{r^2}\,d\tau$$

where

$$r^{-2} \approx r_0^{-2}[1 - 2\lambda_r(1 - \cos\tau)]$$

Table 1 Orbital changes after one-half period of applied in-plane force

	Altitude, Δr	Longitude shift, $\Delta s = r_0\Delta\theta$	Period, ΔT	Eccentricity, e	Longitude oscillation, $2er_0$	Longitude drift rate, per day
Circumferential force						
Before $F_c = 0$	$2\pi\lambda_c r_0$	$-\lambda_c r_0(\frac{3}{2}\pi^2 - 8)$	$3\pi\lambda_c T_0$	$4\lambda_c$	Variable	Variable
After $F_c = 0$	$2\pi\lambda_c r_0$	$-\lambda_c r_0(\frac{3}{2}\pi^2 - 8)$	$3\pi\lambda_c T_0$	$4\lambda_c$	$\pm 8\lambda_c r_0$	$-6\pi^2\lambda_c r_0$
Radial force						
Before $F_r = 0$	$2\lambda_r r_0$	$-2\pi\lambda_r r_0$	$2\lambda_r T_0$	λ_r	$\pm 2\lambda_r r_0$	$-4\pi\lambda_r r_0$
After $F_r = 0$	$2\lambda_r r_0$	$-2\pi\lambda_r r_0$	0	$2\lambda_r$	$\pm 4\lambda_r r_0$	0

Substituting in the preceding integral and performing the integration, gives

$$\Delta\theta = \theta - \tau \approx -2\lambda_r(\tau - \sin\tau) \tag{16}$$

An important property of a constant radial force is that it can be used to sustain the satellite in a circular orbit that has a period equal to that of the force-free orbit and an altitude differing from this orbit. Thus, if T_0 is the period of the force-free, circular orbit and T_1 is the period of the circular orbit under the radial force F_r, then

$$T_0/T_1 = [(\mu'/\mu)(r_0^3/r_1^3)]^{1/2}$$

If $T_0 = T_1$, then $r_1^3 = (\mu'/\mu)r_0^3 = (1 - \lambda_r)r_0^3$. To first-order, this yields

$$\Delta r = r_1 - r_0 \approx -\tfrac{1}{3}\lambda_r r_0 \tag{17}$$

Thus, if $\lambda_r > 0$ ($F_r > 0$), so that F_r is directed away from the earth, then $r_1 < r_0$; if $\lambda_r < 0$ ($F_r < 0$), so that F_r points to the center of the earth, then $r_1 > r_0$.

2.3 Normal Force

Consider a small, constant force F_n to act on the satellite in a direction always normal to the orbital plane (see Fig. 3). Then, if φ denotes the "latitude" of the satellite relative to the plane of the initial orbit, it is not difficult to show that the motion of the satellite is described by

$$\frac{d^2\varphi}{dt^2} + \left(\frac{2\pi}{T_0}\right)^2 \varphi = \frac{1}{r_0} f_n$$

where $f_n = F_n/m$. The solution of this equation is

$$\varphi = \lambda_n(1 - \cos\tau) \tag{18}$$

where

$$\lambda_n = (1/r_0)(T_0/2\pi)^2 f_n \tag{19}$$

Note that φ never becomes negative. The effect of F_n is to tilt the orbital plane an angle α with respect to the original plane where

$$\alpha \approx \tfrac{1}{2}\varphi(\pi) = \lambda_n$$

If F_n is applied for only half an orbit ($\tau = \pi$), the satellite will be at $\varphi = 2\lambda_n$ when F_n is discontinued, so that φ will subsequently oscillate between $\pm 2\lambda_n$ and the orbit will now be inclined at an angle $2\lambda_n$ with the original plane. If F_n is applied alternately positively and negatively for two successive half orbits, the final orbit will be inclined at an angle $4\lambda_n$ with respect to the original plane, and φ will oscillate between $\pm 4\lambda_n$.

2.4 Summary of Orbital Changes

The effect of applying a constant force F_c, F_r, or F_n to an initially circular orbit for a period $T \leq T_0$ can now be summarized. When the circumferential force F_c is discontinued, the subsequent orbit will be an ellipse, except if F_c becomes zero at $\tau = 2\pi$, in which case the orbit will be circular [see Eq. (10)]. When the radial force F_r is discontinued, the subsequent orbit will be an ellipse, except if F_r becomes zero at perigee, i.e., at $\tau = 2\pi$, in which case the orbit will become the original circular orbit of radius r_0. Of particular interest is the behavior of $\Delta\theta$ ($= \theta - \tau$) after F_c or F_r is terminated. For a stationary satellite, $\Delta\theta$ represents the longitudinal motion of the satellite relative to an observer on the equator. Now, the equation of the force-free ellipse is $r^{-1} = \mu h^{-2}(1 + e\cos\theta)$, where $h^2 = \mu a(1 - e^2) \approx \mu a$. Expressing the semimajor axis a in terms of the orbital period T, substituting in $2\pi d\theta/T_0 d\tau = h/r^2$, and performing the integration yields

$$\Delta\theta = \theta - \tau = \tau[(T_0/T) - 1] + 2e\sin\theta \tag{20}$$

Thus, $\Delta\theta$ oscillates with an amplitude $2e$ and period T and also experiences a uniform drift, $\tau[(T_0/T) - 1]$. The drift velocity v_d is

$$v_d = r_0\frac{d}{dt}\left[\tau\left(\frac{T_0}{T} - 1\right)\right] = 2\pi r_0\left(\frac{1}{T} - \frac{1}{T_0}\right) \approx -2\pi r_0\frac{\Delta T}{T_0^2} \tag{21}$$

where $\Delta T = T - T_0$. Note that, if $T = T_0$, then $v_d = 0$. Also, $v_d < 0$ if $T > T_0$, and $v_d > 0$ if $T < T_0$.

The orbital changes Δr, $\Delta\theta$, e, and ΔT, derived previously for the circumferential and radial forces, exist during the application of these forces. After F_c and F_r are terminated, these changes may or may not remain the same. Table 1 shows the orbital changes—in addition to the longitude oscillation and drift velocity—just prior to and just after the force termination for a one-half period ($\tau = \pi$) force application. Table 2 shows these same quantities for a full period ($\tau = 2\pi$) of force action. Table 3 shows the changes in orbit inclination and latitude oscillation due to the normal force F_n for a half period and full period of force.

The results given in the tables are of quite general application to near-circular orbits around any gravitational center,

Table 2 Orbital changes after one period of applied in-plane force

	Altitude, Δr	Longitude shift, $\Delta s = r_0\Delta\theta$	Period, ΔT	Eccentricity, e	Longitude oscillation, $2er_0$	Longitude drift rate per day
Circumferential force						
Before $F_c = 0$	$4\pi\lambda_c r_0$	$-6\pi^2\lambda_c r_0$	$6\pi\lambda_c T_0$	0	0	Variable
After $F_c = 0$	$4\pi\lambda_c r_0$	$-6\pi^2\lambda_c r_0$	$6\pi\lambda_c T_0$	0	0	$-12\pi^2\lambda_c r_0$
Radial force						
Before $F_r = 0$	0	$-4\pi\lambda_r r_0$	$2\lambda_r T_0$	λ_r	$\pm 2\lambda_r r_0$	$-4\pi\lambda_r r_0$
After $F_r = 0$	0	$-4\pi\lambda_r r_0$	0	0	0	0

Table 3 Orbital changes due to normal force

	After one-half period		After one period		After alternately positive and negative for two successive half periods	
	Orbit inclination	Latitude oscillation	Orbit inclination	Latitude oscillation	Orbit inclination	Latitude oscillation
Before $F_n = 0$	λ_n	$+2\lambda_n$ -0	λ_n	$+2\lambda_n$ -0	$3\lambda_n$	$+2\lambda_n$ $-4\lambda_n$
After $F_n = 0$	$2\lambda_n$	$\pm 2\lambda_n$	0	0	$4\lambda_n$	$\pm 4\lambda_n$

the nondimensional accelerations λ_c, λ_r, λ_n being the circumferential, radial, and normal components of thrust acceleration expressed in terms of the gravitational acceleration at distance r_0.

3. Natural Perturbations to the Postion of a Stationary Satellite

3.1 Perturbations Due to the Earth

Perturbations of the stationary satellite attributed directly to the earth are due principally to the earth's nonspherical shape. The exact nature of the earth's electric and magnetic field at the altitude of a 24-hr orbit is not well known, but recent estimates show that they will perturb the satellite with a force less than 10^{-4} times that due to the moon and sun. The forces acting on the satellite due to the asymmetrical earth are derived from the earth's gravitational potential given by

$$V(r, \theta, \varphi) = \frac{-\mu}{r}\left[1 - J_2 \left(\frac{R}{r}\right)^2 P_2 (\sin \varphi) + \ldots - J_2{}^{(2)} \left(\frac{R}{r}\right)^2 P_2{}^{(2)}(\sin\varphi) \cos 2(\theta - \theta_E)\right]$$

Here, r, θ, φ are the polar coordinates of the satellite, defined in Fig. 4, θ_E denotes the earth's equatorial minor axis (Fig. 4), R is the mean equatorial radius of earth, P denotes the Legendre functions, and

$$J_2 = 1.08219(10^{-3}) \qquad J_2{}^{(2)} = 5.35(10^{-6})$$

where J_2 is a first-order measure of the earth's oblateness and $J_2{}^{(2)}$ a measure of the ellipticity of the earth's equatorial section, i.e., of the earth's triaxiality. The force per unit mass acting on the satellite is then $\mathbf{f} = (f_r, f_\theta, f_\varphi) = -\Delta V(r, \theta, \varphi)$, where f_r, f_θ, f_φ are the components of $\mathbf{f}$ that lie in the directions of increasing r, θ, φ, respectively.

Therefore,

$$f_r = \frac{-\partial V}{\partial r} \approx \frac{-\mu}{r^2} + \frac{3}{2}\mu J_2 \frac{R^2}{r^4}(3\sin^2\varphi - 1) + 9\mu J_2{}^{(2)} \frac{R^2}{r^4}\cos^2\varphi \cos 2\gamma$$

$$f_\theta = -\frac{1}{r\cos\varphi}\frac{\partial V}{\partial\theta} \sim 6\mu J_2{}^{(2)} \frac{R^2}{r^4}\cos\varphi \sin 2\gamma$$

$$f_\varphi = -\frac{1}{r}\frac{\partial V}{\partial\varphi} \sim -\frac{3}{2}\mu J_2 \frac{R^2}{r^4}\sin 2\varphi + 3\mu \quad {}^{(2)} \frac{R^2}{r^4}\sin 2\varphi \cos 2\gamma$$

Note that if the orbit is equatorial ($\varphi = 0$), then f_θ and f_φ become the circumferential and normal accelerations, respectively.

The primary effect of f_φ is to cause the orbital plane of the satellite to precess (nodal regression) about the polar axis. The precession is approximately 0.0135 deg/day for a synchronous orbit inclined to the equator at an angle of $0 < i \leq 3°$. The precession is of no consequence to the satellite other than to change the time of day it will cross the equator in a north-south oscillation.

The φ-dependent terms contained in f_r will cause an in-plane perturbation of the orbit, i.e., a precession of the line of apsides, for initially elliptical orbits. The perturbing acceleration is approximately 8.7 (10^{-7}) g's, causing a precession of approximately 0.0134 deg/day. For nearly constant γ, both φ-dependent terms in f_r become constant near the equator ($\varphi = 0$). To obtain a precise 24-hr period, the J_2 term can be compensated by an increase in orbital radius of 1700 ft and the $J_2{}^{(2)}$ term by a decrease of 50 (cos2γ) ft.

A perfect orbital injection may be defined, therefore, as one in which the precise longitude and zero latitude are obtained, with zero velocity, angular velocity, and attitude error in an earth-fixed reference frame, at an altitude differing from the classical synchronous altitude by an amount

$$\delta h = 1700 - 50\ (\cos 2\gamma)\ \text{ft}$$

For a perfect injection, the total perturbing force on the satellite due to the earth's gravitation is therefore f_θ, the circumferential component of f.

The effect of f_θ is to add or subtract satellite orbital energy, depending on the value of $\gamma = \theta - \theta_E$. Thus, if $0 < \gamma < \pi/2$, $\pi < \gamma < \frac{3}{2}\pi$, energy is added to the orbit and the satellite will gain in altitude, and its period will be lengthened. If $-\pi/2 < \gamma < 0$, $\pi/2 < \gamma < \pi$, energy is taken from the orbit, the satellite will lose altitude, and its period will be shortened. The variation of f_θ with longitude is shown in Fig. 5. A posi-

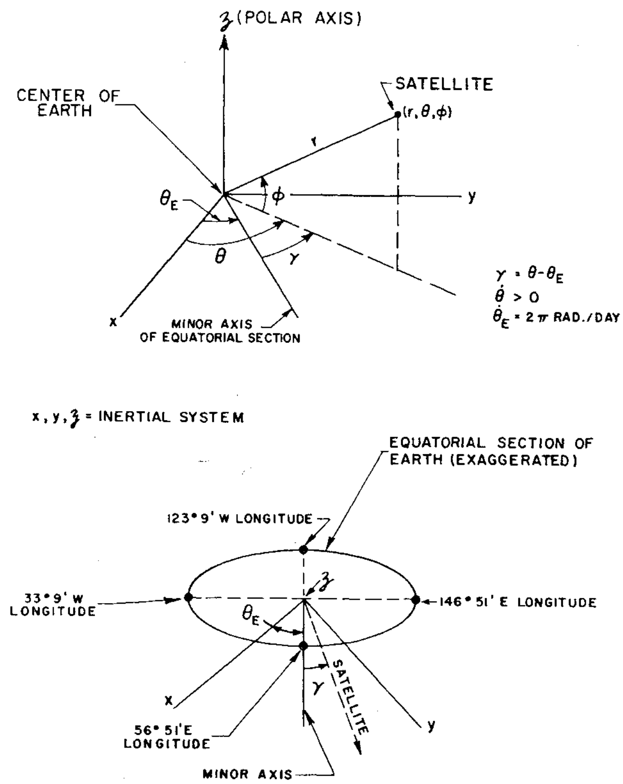

Fig. 4 Definition of r, θ, ϕ; θ_E.

tive (negative) value of f_θ means that f_θ is directed to the east (west). Table 4 shows the values of γ and f_θ at four prospective longitudes.

A longitude of about 90° W is of particular interest because it corresponds to the longitude that would be used for a multiple-access satellite placed in a synchronous orbit for broadcasting purposes. If no corrective forces were applied following perfect injection, then, under the acceleration of $1.54(10^{-8})\ g_0$, the satellite would describe the relative motion shown in Fig. 6. Here $\Delta r = r - r_0$ where $r_0 = 22{,}750$ naut miles and $\Delta s = r_0\alpha$ where $\alpha = \Delta\theta =$ the change in longitude of the satellite. The quantities Δr and $\Delta\theta$ are computed from Eqs. (7) and (8), respectively. The inset in Fig. 6 shows the behavior of Δr and Δs beyond the first day following injection. If no attempt is made to negate f_θ, the satellite will continue to move westward, past the equatorial minor axis, and f_θ will experience a change in sign. The satellite will tend to oscillate about the position 123°9′ W with a period of 1.4 years.[6]

3.2 Perturbations Due to the Moon and Sun

The known extraterrestrial perturbations on the satellite are due to the gravitation of the moon and sun, solar radiation (radiation pressure), Van Allen radiation, cosmic radiation, and meteorite bombardment. Of these, the moon and sun contribute, by far, the most significant perturbations.[6]

The moon-sun perturbations occur both in the plane of the satellite orbit and normal to it. The effect of the in-plane perturbations is to cause a change in orbit altitude not exceeding 1.69 naut miles and a maximum longitudinal shift of

Table 4 f_θ at four prospective longitudes

Longitude	γ	$f\theta$, g's
25° W	−81°51′	$-0.47(10^{-8})$
33°9′ W	−90°	0
78°9′ W	−135°	$1.67(10^{-8})$
90° W	−146°51′	$1.54(10^{-8})$

38.5 naut miles.[6] Thus, for perfect orbit injection, the in-plane perturbations will never cause the satellite to deviate more than about 40 naut miles from its specified position. This corresponds to an angular variation of approximately 7 min of arc as seen from the surface of the earth.

More severe than the in-plane perturbations are those that act normal to the satellite orbit. The out-of-plane perturbations due to the moon and sun cause the orbital plane of the satellite to regress slowly about a line that is nearly normal to the ecliptic. When the sun or moon is overhead at the extreme latitudes (which occur in summer and winter for the sun and twice a month for the moon), the satellite will experience a component of force, normal to the orbit plane, which oscillates with a period of nearly one day. This oscillating force will cause the orbit plane to tilt as shown in Fig. 3, resulting in an inclination that will accumulate at an almost constant rate. The inclination will appear as an oscillation about the equatorial plane. The period of oscillation is approximately one day, and the amplitude of the oscillation increases at a constant rate of 0.8525 deg/yr for the first 10 years.[6] Thus, from a point on the equator, the satellite would appear to oscillate

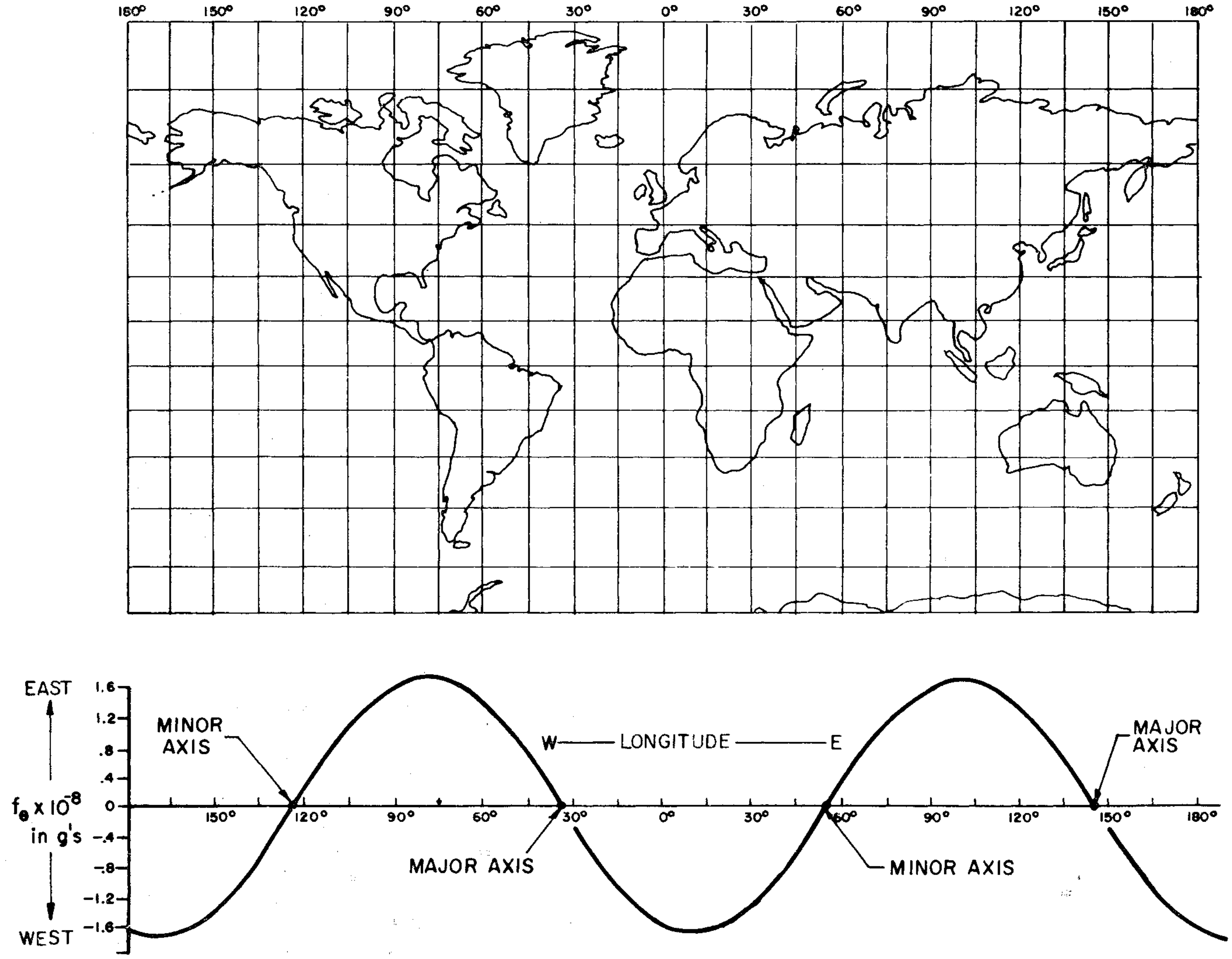

Fig. 5 Perturbation acceleration f_θ vs longitude.

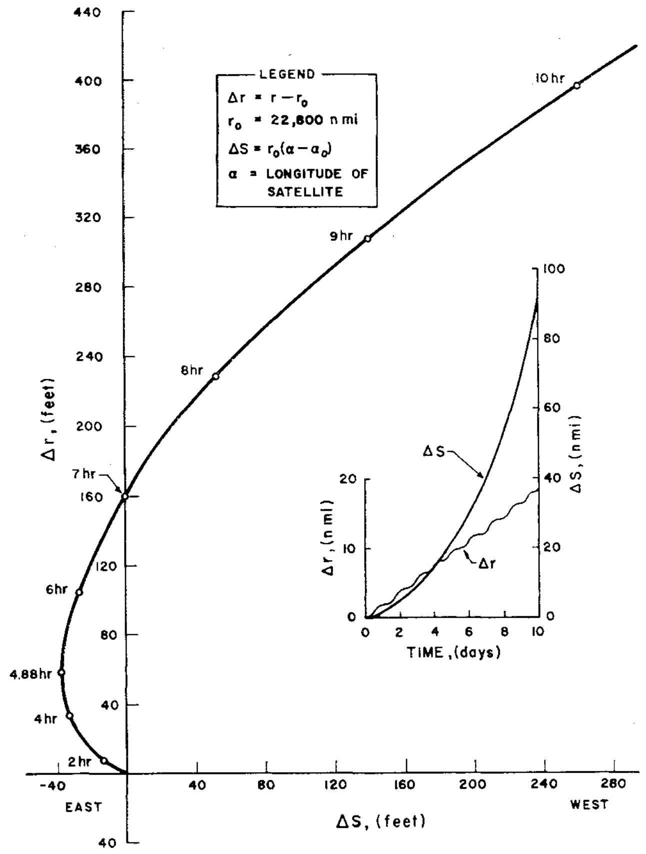

Fig. 6 Relative motion of satellite injected into stationary orbit at 90°W longitude.

daily in a north-south direction with ever-increasing amplitude. Note that within 43 days, the amplitude will already have reached 40 naut miles.

4. Corrections to the Position of a Stationary Satellite

If the stationary satellite has a perfect injection into orbit, it will begin to move relative to an observer on the earth, owing to perturbations caused by the earth's triaxiality and the sun and moon's gravitation. Figure 7c shows the satellite motion in the position window as a consequence of the earth's elliptical equator. This motion is described by Eq. (20). Figure 7d illustrates the motion caused by the sun-moon perturbation. The total motion will be a combination of the previous two figures.

The most direct way of overcoming the effects of the natural perturbations is to have two thrusters onboard the satellite, one continuously thrusting at the correct level in a direction tangent to the orbit and the other thrusting the appropriate amount in a direction normal to the orbit. This method, however, is not feasible, since the perturbations are not accurately known, and even if they were, a control system that would precisely negate them would be extremely difficult to build. The alternative procedure is to turn on one or both of the thrusters whenever significant orbital errors accumulate. The intermittent action of these thrusters will, however, create additional perturbations on the satellite. Thus, errors in altitude, period, longitude, and eccentricity will accrue due to both the natural perturbations and to the corrective maneuvers themselves. Rather than have several on-off thrusting devices attempt to correct the orbital errors, a much simpler and more reliable device would be a single, low-thrust engine, thrusting continually in a nominally radial direction, with the capability of vectoring 1) normal to the orbit and 2) to either side of the radial direction, so as to provide normal and tangential force components to overcome the sun-moon and triaxiality perturbations, respectively. Recall that it is possible for radial thrust to sustain the satellite in a circular orbit having the same period as the stationary orbit (and therefore, itself a stationary orbit), but a different altitude, given by Eq. (17). The engine would also have the capability of applying a torque to the satellite for purposes of attitude control, as discussed later. The thrust would be directed radially for most of the time and would occasionally be rotated a small amount in the appropriate direction to correct the orbital errors. Figure 8 gives a pictorial conception of such a satellite, complete with antenna and solar panels.

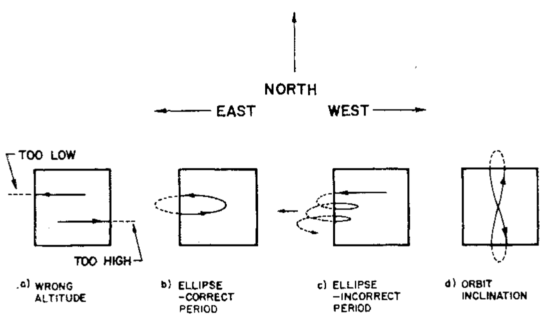

Fig. 7 Motion of stationary satellite in position window.

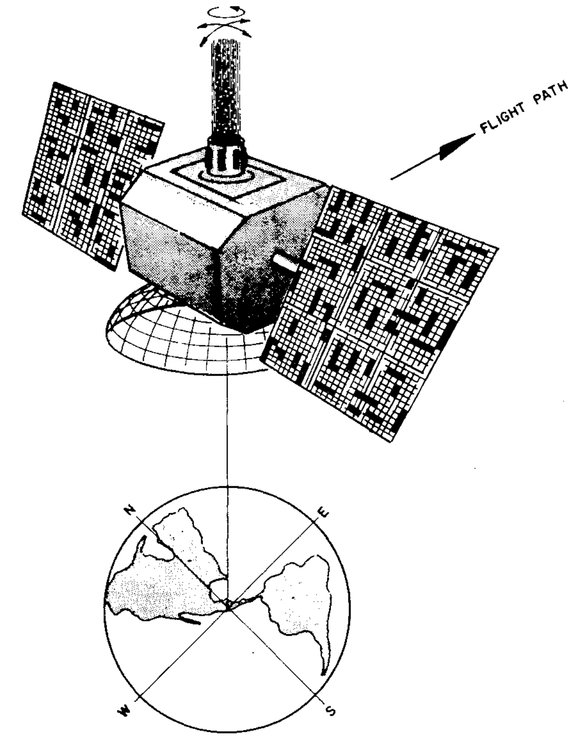

Fig. 8 Satellite using single thruster acting radially inward.

Although the natural perturbations can be overcome only by circumferential (tangential) and normal thrust components, the orbital errors resulting from the intermittent application of these corrective forces can be rectified by both the circumferential and radial thrusts. The orbital errors shown in Tables 1 and 2 resulting from the application of F_c and F_r can, of course, be removed from the orbit by the reverse application of these forces. If radial thrust is being used to sustain the satellite in a stationary orbit, then the radial acceleration in Tables 1 and 2 (i.e., λ_r) refers to the *change* in magnitude of the sustaining radial acceleration.

Of particular interest is the order of magnitude of the circumferential and normal thrust components that would be re-

Table 5 Orbital changes after 12 hr of 1-μg in-plane thrust acceleration

	Altitude change, naut miles	Longitude shift, naut miles	Period change, sec	Eccentricity	Longitude oscillation, naut miles	Longitude drift rate, naut miles/day
Circumferential thrust						
Before $F_c = 0$	6.28	−6.8	35.8	1.76×10^{-4}	Variable	Variable
After $F_c = 0$	6.28	−6.8	35.8	1.76×10^{-4}	±8	−59
Radial thrust						
Before $F_r = 0$	2	−6.28	7.6	4.4×10^{-5}	±2	−12.6
After $F_r = 0$	2	−6.28	0	8.8×10^{-5}	±4	0

quired to overcome the natural perturbations. From Fig. 5 it is readily seen that the required circumferential thrust acceleration is of the order of 2×10^{-8} g. In order to determine the required normal component, we recall that the orbital plane of the stationary satellite oscillates about the equatorial plane with an amplitude that increases at the rate of 0.8525 deg/yr, or 4.07×10^{-5} rad/day. From Table 3, the orbit inclination can be changed by an amount $2\lambda_n$/day by thrusting normal to the orbit for a half day, each day. Thus, $2\lambda_n = 4.07\ (10^{-5})$. For a stationary satellite, $r_0 = 13.83\ (10^7)$ ft, $T_0 = 8.64\ (10^4)$ sec. Hence, from Eq. (19) we find that $f_n = 0.465 \times 10^{-6}\ g$, which is about twenty times the magnitude of the circumferential component. From Table 3 we see that the sun-moon perturbation could also be overcome by thrusting alternately positively and negatively for two successive 12-hr intervals. In this case, $4\lambda_n = 4.07\ (10^{-5})$, so that f_n would be only $0.233 \times 10^{-6}\ g$. For a 550-lb satellite,‡ the normal thrust component would then be either 0.256 mlb (millipound) or 0.128 mlb, depending upon the method of thrusting. If the nominal radial thrust (i.e., sustaining thrust) is 2.5 mlb,‡ then this thrust would have to be rotated by either 5.86° or 2.93° to correct the orbit perturbation. For a nominal radial thrust of 2.5 mlb, Eq. (17) shows that the altitude of the sustained orbit differs from that of the nonthrusted stationary orbit by 1.52 naut miles.

In view of the preceding, it appears that the magnitude of the circumferential, normal, and incremental radial thrust acceleration for purposes of general orbit correction will be of the order of magnitude of 1 μ g. Tables 5–7 show the orbital changes in Tables 1–3 for the case of a stationary orbit employing this value of thrust acceleration, i.e., $\lambda_c = \lambda_r = \lambda_n = 4.4 \times 10^{-5}$. Note that since all entries in these tables are linear in λ, they can be scaled for other values of acceleration and are accurate to better than 1% for accelerations less than 100 μg.

5. Attitude Control

The power levels required by the second generation stationary satellites will be too large to permit them to be spin stabilized. Hence, these satellites will require an attitude control system to provide orientation of their solar panels, or solar collectors, and to keep the satellite antenna pointing in the direction of the earth. The classical solution to the attitude control problem is to employ a system of twelve reaction jets, four for each rotation axis. These thrusters would be used in pairs to provide pure couples for attitude correction and paired differentially to provide translational motion. In practice, however, this system suffers a number of defects. Among the more conspicuous ones are the errors in the attitude sensors, the slow response of the actuators, and the errors in matching the magnitude and duration of the thrust delivered by the pairs providing translation. Employing the single, *radially thrusting* engine for attitude control avoids many of the conspicuous errors of the multiple thruster system.

The radial thruster can provide attitude control in pitch and roll simply by mounting it on a rigid lever arm and rotating the line of thrust away from the satellite center of mass, so as to provide a torque on the satellite. Thus, the thrust would normally be directed through the center of mass, unless the satellite acquired an attitude error, in which case the thruster would be rotated by the proper amount to cancel out the error. The yaw attitude requires a different form of control which can be accomplished by providing vorticity in the thruster exhaust. For gasjets or arcjets this can be effected by vanes and for ion engines by electrode movement.

The demands placed on this attitude control system due to natural perturbations should be very slight. The most significant natural perturbations of the satellite attitude are due to 1) gravity gradient, 2) solar radiation, 3) uneven surface heating, and 4) meteorites. Proper design and construction of the satellite will greatly minimize the first three and perhaps the last. Since the direction of thrust cannot be permanently changed without rotating the satellite, the most important function of this control system will be to rotate the satellite so as to provide a component of thrust normal to the orbital plane in order to overcome the north-south perturbation due to the sun and moon described earlier. To show how this is done consider the case when the sustaining radial thrust is directed toward the center of the earth, and suppose that the satellite has drifted to the north. The thruster first would be vectored to the south (to produce thrust in the northerly direction), thereby producing a torque about the pitch axis (i.e., the flight direction). This torque would persist until the satellite had rotated to the south through some specified angle, e.g., 3°. The thrust vector would then be rotated in the opposite direction so as to provide a counteracting torque about the pitch axis, thereby arresting the induced rotation, with the satellite inclined at twice the initial deflection (i.e., 2 × 3°). At this instant, the thrust vector would be aligned through the satellite center of mass and would pass through the earth's rotational axis at a point *below* the earth's center

Table 6 Orbital changes after 24 hr of 1-μg in-plane thrust acceleration

	Altitude change, naut miles	Longitude shift, naut miles	Period change, sec	Eccentricity	Longitude oscillation, naut miles	Longitude drift rate, naut miles/day
Circumferential thrust						
Before $F_c = 0$	12.6	−59	71.6	0	0	Variable
After $F_c = 0$	12.6	−59	71.6	0	0	−118
Radial thrust						
Before $F_r = 0$	0	−12.6	7.6	4.4×10^{-5}	±2	− 12.6
After $F_r = 0$	0	−12.6	0	0	0	0

‡ Chosen for comparison with the NASA Lewis satellite control system.

Table 7 Orbital changes due to 1-μg normal thrust acceleration

	After 12 hr		After 24 hr		After alternately positive and negative for two successive half periods	
	Orbit inclination, deg	Latitude oscillation, naut miles	Orbit inclination, deg	Latitude oscillation, naut miles	Orbit inclination, deg	Latitude oscillation, naut miles
Before $F_n = 0$	2.52×10^{-3}	$+2$ -0	2.52×10^{-3}	$+2$ -0	7.56×10^{-3}	$+2$ -4
After $F_n = 0$	5.04×10^{-3}	± 2	0	0	10^{-2}	4

of mass, thereby providing a component of thrust normal to the orbital plane.

Since an important function of the stationary satellite control is to keep an antenna or camera pointing at the earth, rotating the entire satellite including the antenna in order to generate a component of thrust normal to the orbit could be a serious objection to this control system. Of course, the greater the nominal radial thrust, the smaller will be the rotation needed to generate a given normal force. It was shown previously that by thrusting (in a direction normal to the orbit) alternately positively and negatively for two successive 12-hr intervals, the thrust direction, and hence the satellite, would have to be rotated through 2.93° in order to overcome the sun-moon perturbation, assuming a nominally radial thrust of 2.5-mlb and a 550-lb satellite. If the satellite is a multiple access communication satellite, then the tolerances in attitude might be sufficient to allow a rotation of 2.93°. For an attitude control system having zero tolerance, the cone angle for the antenna would be 17° (the angle subtended by the earth at the satellite); but in order to insure continuous coverage of the earth with a tolerance of ±2.93°, the cone angle would have to be increased to $17° + 2 \times 2.93° = 22.86°$. Thus, to provide the same signal strength over the receiving hemisphere, it would also be necessary to increase the antenna gain by

$$[(22.86/17)^2 - 1] \times 100 = 81\%$$

For sufficiently high antenna gain and/or large radial thrusts, this attitude control system would be suitable for multiple access communication satellites.

For a point-to-point communication, or reconnaissance/surveillance satellite, a rotation of 2.93° cannot be tolerated, since the narrow beam would shift a distance of 1160 naut miles across the earth's surface. In order to use the radial thruster concept for the more accurately pointed satellite, or even for a multiple access communication satellite using a low radial thrust and moderate antenna gain, it will be necessary to provide a means for rotating the thrust vector without rotating the satellite. A method for doing this is shown in Fig. 9. Here, the thruster is mounted on an arm that is gimballed at two points instead of one, the second point being the satellite center of mass. Thus, rotating the arm about the center of mass will change the direction of thrust without rotating the satellite and thereby provide position control to the satellite. Rotating the thruster about the outer gimbal will provide the small corrections in attitude. Since the required deflections will be small, both gimbals can be simple flexure pivots. The accuracy of this control system could be made extremely high if accurate attitude and attitude-rate sensors were used to dictate the gimbal angle by proportional control. Separation of the position and attitude control modes of operation by this method is somewhat sophisticated, but has an advantage over a multiple thruster system in that corrections may always be applied simultaneously, rather than sequentially.

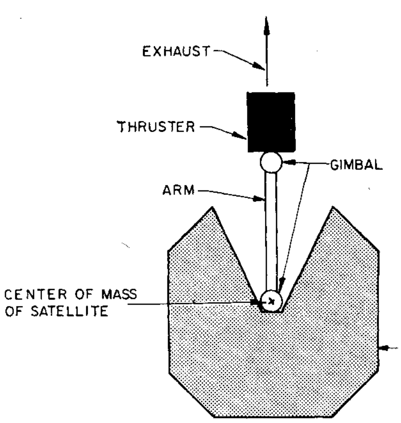

Fig. 9 Position and attitude control system for stationary satellite.

6. Use of Ion Propulsion

Ion propulsion is particularly suited to the task of providing position and attitude control to a stationary satellite for which a long lifetime is anticipated. Ion propulsion is also being considered for thrusters that would be used intermittently in the same way as the cold gasjet. The characteristics and operation of ion engines are covered in great detail elsewhere.[7] In principle, they employ electrical energy to accelerate ionized particles to extremely high velocities, giving a large total impulse for a small consumption of propellant.

6.1 Multiple Thruster Approach

Many configurations are possible in which multiple fixed or movable thrusters are used to provide the functions of position and attitude control of a stationary satellite. The system under development for NASA-Lewis[8] uses nine fixed thrusters (in three clusters of three), of which three are used for position control and six for attitude control. The system weight for the thrusters and power conditioning equipment is kept below 125 lb, not including the power source, power storage, nor attitude sensing and computing systems.

The thrusters are to be used on a maximum duty cycle (for using electrical power) of 5%. The cycle time will be determined by the attitude control accuracy required, since most system designs will incorporate a limit cycle ranging from about 20 min to 1 hr about one or more axes. The thrusters must be capable of fast turn-on and shut-down if energy and propellant are both to be conserved, since the process of energizing a thruster for a brief attitude control correction is particularly wasteful.

6.2 Radial Thruster Approach

Continuum-flow thrusters such as chemical rockets and arcjets can be provided with vector control about all three axes by deflection vanes or by fluid injection techniques. Experimental evidence indicates that multiple-aperture contact and bombardment ion engines are also capable of vector control about all three axes.[7,9] The exhaust beam may be deflected to either side of its nominal direction by a lateral movement of the accelerating electrode, thereby providing control in pitch and roll. Control in yaw is provided by rotating the electrode; this imparts a helical motion to the beam, thereby creating a torque about the yaw axis.

The successful application of ion thrusters to the synchronous satellite does not depend, however, on the feasibility of electrostatic deflection of the beam. If mechanical actuation is to be used, it will be necessary to use two thrusters instead of a single module so that differential deflection can produce torque about the thrust axis.

In order to compare the single radial thruster with the nine-thruster system of the 3-yr NASA satellite, we consider a 550-lb satellite and a radial thrust level of 2.5 mlb. Conservative weight estimates for the total thrusting systems of both types are given in Table 8, and it will be seen that the weights are quite comparable. The extra weight of propellant and solar panels of the single thruster system is offset by the total weight of the other engines and by the lack of necessity for carrying batteries. It must be emphasized that neither of these systems is optimum for the 550-lb satellite, but they are sufficiently representative for the similarity in weight to be considered noteworthy.

7. Merits of the Radial Thruster Approach

7.1 Simplicity

If we confine our system to a constant radial thrust directed either toward or away from the earth, we need only to command positive and negative rotation of the satellite about three axes, whereas a conventional system would require additional commands for translational thrust in at least three directions. The installation of the single thruster within the satellite can be done with less complexity than with the multiple thruster.

With a sufficiently sensitive ground-based position control, it may be possible to eliminate the attitude control system entirely from the satellite. If the translational velocities are kept small enough, the attitude control will follow as a consequence of position control, and attitude sensors will not be required. A careful analysis of artificial damping and of the capabilities and inherent errors of sensors will permit the need for attitude control in a given design to be properly assessed.

In addition, the use of continuous rather than intermittent thrust allows for much simpler design of the engine control system, the propellant feed system, and the power conditioning equipment.

Table 8 Weight summary for ion thruster systems

Thruster assembly	Nine 1.5-mlb engines[a] Number	Nine 1.5-mlb engines[a] Total weight, lb	One 2.5-mlb engine[b] weight, lb
Engine, reservoir and feed	9	40.5	6
Propellant for 3 years	9	4.5[a]	36[b]
Ionizer and vaporizer heater transformers	9	22.5	3
Engine Mounting plate	3	9	2
Power Conditioning			
High voltage supply	1	12	14
Ionizer heater supply	1	5[a]	4[b]
Vaporizer supply	1	3[a]	2[b]
Misc. heater and bias supply	1	4	4
Master power control and switches	1	4[a]	2[b]
Activators and sundries			
Activators and support structure	1	...	4
Structural clamps	6	2.5	2
Interconnect blocks	4	3	2
Harness, cabling and miscellaneous	...	9	3
Power source			
Solar panels	2	10	93
Batteries	1	45	...[c]
Orientation system	1	15	15
		189 lb	192 lb

[a] Intermittent rating.
[b] Continuous rating.
[c] The thruster need not be used in the earth's shadow. The orbit errors arising can be readily corrected.

7.2 Low Interference

The exhaust gas from an electric thruster, and even from a chemical thruster, will consist of large numbers of positive and negative ions that will form an essentially neutral plasma beam extending for large distances in space. Little is known yet about the behavior of such an extensive plasma with regard to communications, and it is to be hoped that any deleterious effects on communications will be extremely small.

However, in the event that the exhaust plasma does interfere with communications, there appear to be two possibilities that make the radial thruster concept attractive. First, if the interference is wide band rf noise, this could seriously affect not only transmission from the satellite, but the receiving of command signals from the ground. The directionality that such interference would possess is not well known, but it seems most likely that mounting the thruster to thrust radially inward, as shown in Fig. 8, would minimize this interference.

Second, if the interference provided by the engine tends to be narrow band, or even wide band, but transmission appears to have a high gain along the beam direction, then there are a number of ways in which the beam itself may conceptually be used for transmission purposes, eliminating the need for the usual antenna.

7.3 High Reliability

The various factors that determine the reliability of a device include considerations of degradation rate, mean-times-to-failure, and number-of-cycles-to-failure. Design for high reliability requires the acknowledgment of degradation rates in providing additional performance capabilities and acknowledgment of potential failures by providing suitable redundancy. For system control using multiple thrusters there may well be a requirement for redundant thrusters at each location, and, if the intermittent operation requires intermittent use of a valve, special provisions must be made for valves that may stick open or closed.

By a single radial thruster approach, we understand the use of a thruster unit capable of vector control together with capability for torquing about its own axis. We also recognize that this approach may involve the provision of two engines instead of a single engine if mechanical actuation is to be used. However, whether the thruster be a single unit or a pair of parallel units, the system will have inherently higher reliability than a nonredundant multiple thruster system in which every engine must work. With only one or two thrusters, we can more readily design for high reliability by constructing the engines on a modular basis, so that sufficient reliability is insured by the capability of operating with some modules not functioning. The provision of an extra thruster assembly, with means for its substitution, might also be considered for little change in system weight.

In addition, we should expect in practice that the reliability for a device operating continuously is inherently higher than that for a device operating intermittently, especially since numbers of cycles as high as 250,000 are under consideration for the multiple thruster approach.

7.4 Initial Station-Seeking Capabilities

An inherent advantage of a position and attitude control system that operates by performing navigation maneuvers in a force field is that it is readily adaptable to the problem of seeking its initial station, thereby reducing the demand for high precision in injection. Since communications during the station-seeking maneuver need not require correct orientation of the multiple access transmitting antenna, the whole satellite can be oriented so that the primary thruster is aligned close to the design flight path, and the thrust program to be applied will be similar to that for an orbit injection scheme with deflecting jets. The power that is normally used for communications may possibly be diverted to increase the specific impulse of the electric thruster.

It is possible to use an electric thruster not only to seek and adjust the terminal orbit of the satellite, but to raise the satellite from a lower orbit into the synchronous orbit. Further, in the event that the position control system breaks down temporarily, it will be possible to correct for very large drifts by repeating the station-seeking maneuver when control is regained.

8. Summary

A description is given of the development of a consistent set of formulas which describes the motion of a satellite in an arbitrary circular orbit, resulting from the action of small, continuous forces acting circumferentially, radially, and normally to the orbit for short time periods. It is assumed that the thrust accelerations are small compared with the acceleration due to gravity. Under this assumption, the formulas are derived by the method of expansion in terms of small quantities and the resulting expressions are thus linear functions of the thrust acceleration. Previous analyses have yielded results that are not as readily applicable to the general motion under an arbitrary small acceleration.

The formulas are then employed to describe the motion of an equatorial synchronous satellite that is undergoing corrective orbital maneuvers by a single thruster, nominally pointing in a radial direction, but vectored to produce circumferential, normal, and incremental radial thrust components. It is shown that the single thruster can perform all the functions of position and attitude control if it also has the capability of producing torque about the thrust axis.

A scheme for implementing the concept of control of the position and attitude of a stationary satellite using an ion engine is briefly evaluated and is found to be generally competitive with the multiple thruster approach now under development for long-life satellites.

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